UNIT 2 LESSON 8

SOLVING BY COMPLETING THE SQUARE

*Completing the square is a technique we use to create perfect square trinomial in order to make a quadratic factorable. When you can't perform A*C to find B → Complete the Square!!!

Completing the Square to Solve Quadratic Equations of the Form $ax^2 + bx + c = 0$
1. Make sure the equation is in standard form, $ax^2 + bx + c = 0$.
2. Subtract c from both sides of the equation.
3. If a is not equal to 1, divide each term by a to get a leading coefficient of 1.
4. Add the square of one-half of b to both sides to complete the square.
5. Express the perfect square trinomial as the square of a binomial.
6. Solve by taking the square root of both sides of the equation.

Ex 1) Find the value of c so that the expression is a perfect square trinomial.

$x^2 + 12x + c$

Step 1) Take half of 'b' and square it $\left(\frac{b}{2}\right)^2$

b = 12 → half of b = 6 → square it = $6^2 = 36$

OR

$\left(\frac{12}{2}\right)^2 = 36$

ANSWER: $x^2 + 12x + 36$

Ex 2) Solve $x^2 - 8x + 16 = 4$ by completing the square.

Step 1) Determine if $x^2 - 8x + 16$ is a perfect square trinomial.

Take half of the value of b and then square the result. If this is equal to the value of c, then the expression is a perfect square trinomial.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-8}{2}\right)^2 = 16$$

$x^2 - 8x + 16$ is a perfect square trinomial because the square of half of -8 is 16.

Step 2) Write the left side of the equation as a binomial squared.

Half of b is -4, so the left side of the equation can be written as $(x-4)^2$.

$(x-4)^2 = 4$

Step 3) Isolate x.

$(x-4)^2 = 4$	Perfect square trinomial
$x - 4 = \pm 2$	Take the square root of both sides.
$x = 4 \pm 2$	Add 4 to both sides.
$x = 4 + 2 = 6$ or $x = 4 - 2 = 2$	Split the answer into two separate equations and solve for x.

Step 4) Determine the solution(s).

The equation has two solutions, $x = 2$ or $x = 6$.

Ex 3) Solve $5x^2 - 50x - 120 = 0$ by completing the square.

Step 1) Determine if $5x^2 - 50x - 120 = 0$ is a perfect square trinomial.

The leading coefficient is not 1.

First divide both sides of the equation by 5 so that $a = 1$.

$5x^2 - 50x - 120 = 0$	Original equation
$x^2 - 10x - 24 = 0$	Divide both sides by 5.

Now that the leading coefficient is 1, take half of the value of b and then square the result. If the expression is equal to the value of c, then it is a perfect square trinomial.

$$\left(\frac{b}{2}\right)^2 = \left(\frac{-10}{2}\right)^2 = 25$$

$5x^2 - 50x - 120 = 0$ is not a perfect square trinomial because the square of half of -10 is not -24.

Step 2) Complete the square.

$x^2 - 10x - 24 = 0$	Equation
$x^2 - 10x = 24$	Add 24 to both sides.
$x^2 - 10x + (-5)^2 = 24 + (-5)^2$	Add the square of half of the coefficient of the x-term to both sides to complete the square.
$x^2 - 10x + 25 = 49$	Simplify.

Step 3) Express the perfect square trinomial as the square of a binomial.

Half of b is -5, so the left side of the equation can be written as $(x-5)^2$.

$$(x-5)^2 = 49$$

Step 4) Isolate x.

$(x-5)^2 = 49$	Equation
$x - 5 = \pm\sqrt{49} = \pm 7$	Take the square root of both sides.
$x = 5 \pm 7$	Add 5 to both sides.
$x = 5 + 7 = 12$ or $x = 5 - 7 = -2$	Split the answer into two separate equations and solve for x.

Step 5) Determine the solution(s).

The equation $5x^2 - 50x - 120 = 0$ has two solutions, $x = -2$ or $x = 12$.