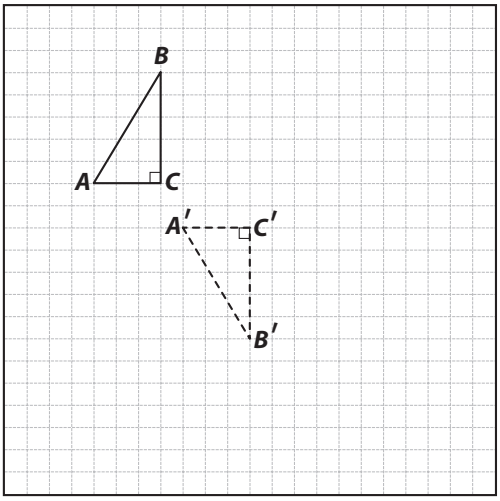
Guided Practice 5.5.2

Example 1

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical legs, count the number of units for the length. For the hypotenuse, use the Pythagorean Theorem, $a^2 + b^2 = c^2$, for which a and b are the legs and c is the hypotenuse.

$AC = 3$	$A'C' = 3$
$CB = 5$	$C'B' = 5$
$AC^2 + CB^2 = AB^2$	$A'C'^2 + C'B'^2 = A'B'^2$
$3^2 + 5^2 = AB^2$	$3^2 + 5^2 = A'B'^2$
$34 = AB^2$	$34 = A'B'^2$
$\sqrt{34} = \sqrt{AB^2}$	$\sqrt{34} = \sqrt{A'B'^2}$
$AB = \sqrt{34}$	$A'B' = \sqrt{34}$

   The sides in the first triangle are congruent to the sides of the second triangle. *Note*: When taking the square root of both sides of the equation, reject the negative value since the value is a distance and distance can only be positive.

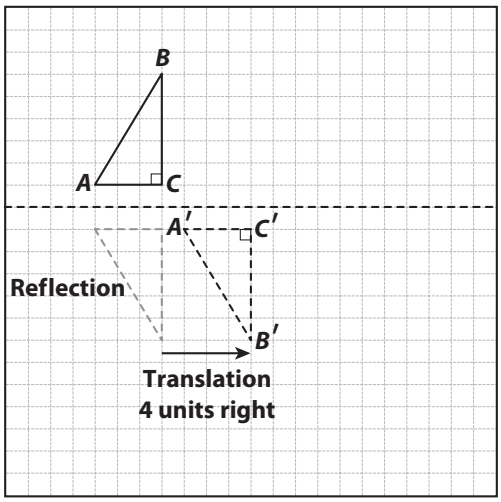
Instruction

2. Identify the transformations that have occurred.

   The orientation has changed, indicating a rotation or a reflection.

   The second triangle is a mirror image of the first, but translated to the right 4 units.

   The triangle has undergone rigid motions: reflection and translation.



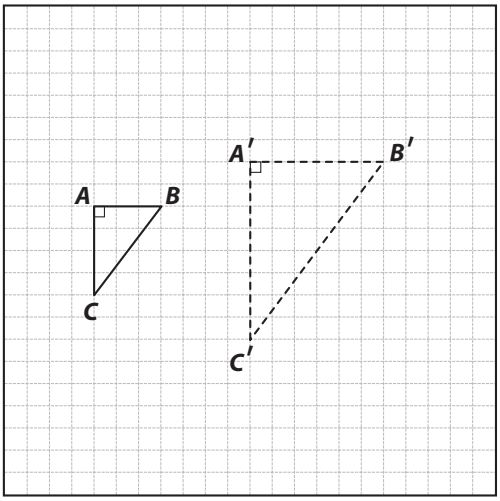
3. State the conclusion.

   The triangle has undergone two rigid motions: reflection and translation. Rigid motions preserve size and shape. The triangles are congruent.

Instruction

Example 2

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical legs, count the number of units for the length. For the hypotenuse, use the Pythagorean Theorem, $a^2 + b^2 = c^2$, for which a and b are the legs and c is the hypotenuse.

$AB = 3$	$A'B' = 6$
$AC = 4$	$A'C' = 8$
$AB^2 + AC^2 = CB^2$	$A'B'^2 + A'C'^2 = C'B'^2$
$3^2 + 4^2 = CB^2$	$6^2 + 8^2 = C'B'^2$
$25 = CB^2$	$100 = C'B'^2$
$\sqrt{25} = \sqrt{CB^2}$	$\sqrt{100} = \sqrt{C'B'^2}$
$CB = \sqrt{25}$	$C'B' = \sqrt{100}$
$CB = 5$	$C'B' = 10$

   The sides in the first triangle are not congruent to the sides of the second triangle. They are not the same size.

Instruction

2. Identify the transformations that have occurred.

   The orientation has stayed the same, indicating translation, dilation, stretching, or compression. The vertical and horizontal distances have changed. This could indicate a dilation.

3. Calculate the scale factor of the changes in the side lengths.

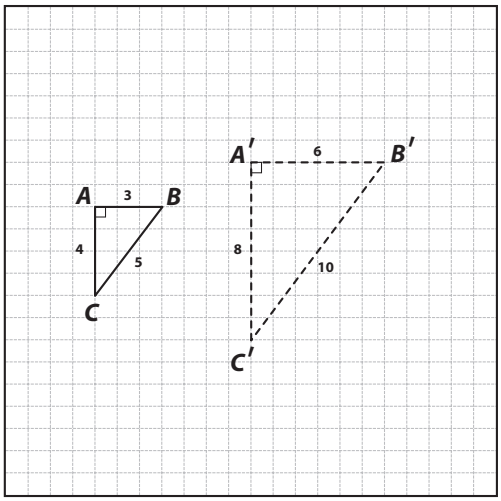
   Divide the image side lengths by the preimage side lengths.

   $$\frac{A'B'}{AB} = \frac{6}{3} = 2$$

   $$\frac{A'C'}{AC} = \frac{8}{4} = 2$$

   $$\frac{C'B'}{CB} = \frac{10}{5} = 2$$

   The scale factor is constant between each pair of sides in the preimage and image. The scale factor is 2, indicating a dilation. Since the scale factor is greater than 1, this is an enlargement.



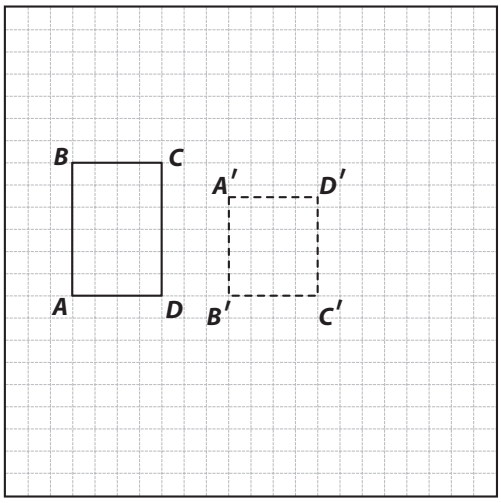
4. State the conclusion.

   The triangle has undergone at least one non-rigid motion: a dilation. Specifically, the dilation is an enlargement with a scale factor of 2. The triangles are not congruent because dilation does not preserve the size of the original triangle.

Instruction

Example 3

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical sides, count the number of units for the length.

   $AB = 6$ $A'B' = 4.5$
   $BC = 4$ $B'C' = 4$
   $CD = 6$ $C'D' = 4.5$
   $DA = 4$ $D'A' = 4$

   Two of the sides in the first rectangle are not congruent to two of the sides of the second rectangle. Two sides are congruent in the first and second rectangles.

Instruction

2. Identify the transformations that have occurred.

The orientation has changed, and two side lengths have changed. The change in side length indicates at least one non-rigid motion has occurred. Since not all pairs of sides have changed in length, the non-rigid motion must be a horizontal or vertical stretch or compression.

The image has been reflected since $\overline{BC}$ lies at the top of the preimage and $\overline{B'C'}$ lies at the bottom of the image. Reflections are rigid motions. However, one non-rigid motion makes the figures not congruent. A non-rigid motion has occurred since not all the sides in the image are congruent to the sides in the preimage.

The vertical lengths have changed, while the horizontal lengths have remained the same. This means the transformation must be a vertical transformation.

Instruction

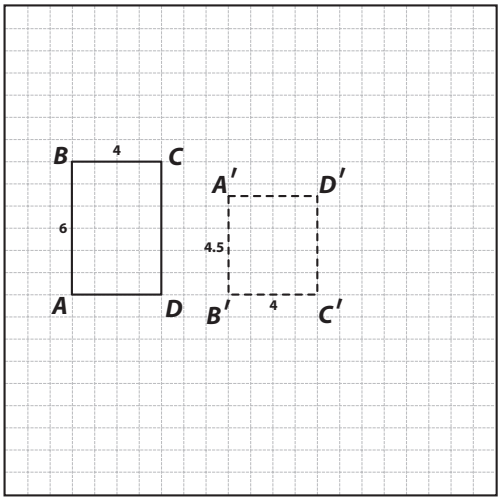
3. Calculate the scale factor of the change in the vertical sides.

   Divide the image side lengths by the preimage side lengths.

$$\frac{A'B'}{AB}=\frac{4.5}{6}=0.75$$

$$\frac{C'D'}{CD}=\frac{4.5}{6}=0.75$$

   The vertical sides have a scale factor of 0.75. The scale factor is between 0 and 1, indicating compression. Since only the vertical sides changed, this is a vertical compression.



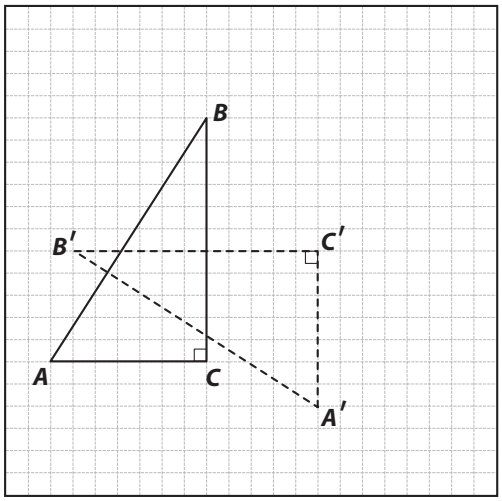
4. State the conclusion.

   The vertical sides of the rectangle have undergone at least one non-rigid transformation of a vertical compression. The vertical sides have been reduced by a scale factor of 0.75. Since a non-rigid motion occurred, the figures are not congruent.

Instruction

Example 4

Determine if the two figures below are congruent by identifying the transformations that have taken place.



1. Determine the lengths of the sides.

   For the horizontal and vertical sides, count the number of units for the length. For the hypotenuse, use the Pythagorean Theorem, $a^2 + b^2 = c^2$, for which a and b are the legs and c is the hypotenuse.

$BC = 11$	$B'C' = 11$
$CA = 7$	$C'A' = 7$
$BC^2 + CA^2 = AB^2$	$B'C'^2 + C'A'^2 = A'B'^2$
$11^2 + 7^2 = AB^2$	$11^2 + 7^2 = A'B'^2$
$170 = AB^2$	$170 = A'B'^2$
$\sqrt{170} = \sqrt{AB^2}$	$\sqrt{170} = \sqrt{A'B'^2}$
$AB = \sqrt{170}$	$A'B' = \sqrt{170}$

   The sides of the first triangle are congruent to the sides of the second triangle.

Instruction

2. Identify the transformations that have occurred.

   The orientation has changed and all side lengths have stayed the same. This indicates a reflection or a rotation. The preimage and image are not mirror images of each other. Therefore, the transformation that occurred is a rotation.

3. State the conclusion.

   Rotations are rigid motions and rigid motions preserve size and shape. The two figures are congruent.