UNIT 5 • CONGRUENCE, PROOF, AND CONSTRUCTIONS
Lesson 5: Exploring Congruence

## Guided Practice 5.5.1

## Example 1

Describe the transformation that has taken place in the diagram below.


1. Examine the orientation of the figures to determine if the orientation has changed or stayed the same. Look at the sides of the triangle.

| Side length | Preimage orientation | Image orientation |
| :--- | :--- | :--- |
| Shortest | Bottom of triangle and <br> horizontal | Bottom of triangle and <br> horizontal |
| Longest | Right side of triangle <br> going from top left to <br> bottom right | Right side of triangle <br> going from top left to <br> bottom right |
| Intermediate | Left side of triangle and <br> vertical | Left side of triangle and <br> vertical |

The orientation of the triangles has remained the same.
2. Connect the corresponding vertices with lines.


The lines connecting the corresponding vertices appear to be parallel.
3. Analyze the change in position.

Check the horizontal distance of vertex $A$. To go from $A$ to $A^{\prime}$ horizontally, the vertex was shifted to the right 6 units. Vertically, vertex $A$ was shifted down 5 units. Check the remaining two vertices. Each vertex slid 6 units to the right and 5 units down.


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## Example 2

Describe the transformation that has taken place in the diagram below.


1. Examine the orientation of the figures to determine if the orientation has changed or stayed the same. Look at the sides of the figures and pick a reference point. A good reference point is the outer right angle of the figure. From this point, examine the position of the "arms" of the figure.

| Arm | Preimage orientation | Image orientation |
| :---: | :--- | :--- |
| Shorter | Pointing upward from the <br> corner of the figure with <br> a negative slope at the <br> end of the arm | Pointing downward from <br> the corner of the figure <br> with a positive slope at <br> the end of the arm |
| Longer | Pointing to the left from <br> the corner of the figure <br> with a positive slope at <br> the end of the arm | Pointing to the left from <br> the corner of the figure <br> with a negative slope at <br> the end of the arm |

(continued)

The orientation of the figures has changed. In the preimage, the outer right angle is in the bottom right-hand corner of the figure, with the shorter arm extending upward. In the image, the outer right angle is on the top righthand side of the figure, with the shorter arm extending down.

Also, compare the slopes of the segments at the end of the longer arm. The slope of the segment at the end of the arm is positive in the preimage, but in the image the slope of the corresponding arm is negative. A similar reversal has occurred with the segment at the end of the shorter arm. In the preimage, the segment at the end of the shorter arm is negative, while in the image the slope is positive.
2. Determine the transformation that has taken place.

Since the orientation has changed, the transformation is either a reflection or a rotation. Since the orientation of the image is the mirror image of the preimage, the transformation is a reflection. The figure has been flipped over a line.
3. Determine the line of reflection.

Connect some of the corresponding vertices of the figure. Choose one of the segments you created and construct the perpendicular bisector of the segment. Verify that this is the perpendicular bisector for all segments joining the corresponding vertices. This is the line of reflection.
The line of reflection for this figure is $y=-1$.


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## Example 3

Describe the transformation that has taken place in the diagram below.


1. Examine the orientation of the figures to determine if the orientation has changed or stayed the same.

Look at the sides of the triangle.

| Side length | Preimage orientation | Image orientation |
| :--- | :--- | :--- |
| Shortest | Right side of triangle and <br> vertical | Top of triangle and <br> horizontal |
| Longest | Diagonal from bottom <br> left to top right of <br> triangle | Diagonal from top left to <br> bottom right of triangle |
| Intermediate | Bottom side of triangle <br> and horizontal | Right side of triangle and <br> vertical |

The orientation of the triangles has changed.
2. Determine the transformation that has taken place.

Since the orientation has changed, the transformation is either a reflection or a rotation. Since the orientation of the image is NOT the mirror image of the preimage, the transformation is a rotation. The figure has been turned about a point. All angles that are made up of the preimage vertex to the reflection point to the corresponding image vertex are congruent. This means that $\angle A R A^{\prime} \cong \angle B R B^{\prime} \cong \angle C R C^{\prime}$.


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## Example 4

Rotate the given figure $45^{\circ}$ counterclockwise about the origin.


1. Create the angle of rotation for the first vertex.

Connect vertex $A$ and the origin with a line segment. Label the point of rotation $R$. Then, use a protractor to measure a $45^{\circ}$ angle. Use the segment from vertex $A$ to the point of rotation $R$ as one side of the angle. Mark a point $X$ at $45^{\circ}$. Draw a ray extending out from point $R$, connecting $R$ and $X$. Copy $\overline{A R}$ onto $\overrightarrow{R X}$. Label the endpoint that leads away from the origin $A^{\prime}$.


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2. Create the angle of rotation for the second vertex.

Connect vertex $B$ and the origin with a line segment. The point of rotation is still $R$. Then, use a protractor to measure a $45^{\circ}$ angle. Use the segment from vertex $B$ to the point of rotation $R$ as one side of the angle. Mark a point $Y$ at $45^{\circ}$. Draw a ray extending out from point $R$, connecting $R$ and $Y$. Copy $\overrightarrow{B R}$ onto $\overrightarrow{R Y}$. Label the endpoint that leads away from the origin $B^{\prime}$.

3. Create the angle of rotation for the third vertex.

Connect vertex $C$ and the origin with a line segment. The point of rotation is still $R$. Then, use a protractor to measure a $45^{\circ}$ angle. Use the segment from vertex $C$ to the point of rotation $R$ as one side of the angle. Mark a point $Z$ at $45^{\circ}$. Draw a ray extending out from point $R$, connecting $R$ and $Z$ and continuing outward from $Z$. Copy $\overline{C R}$ onto $\overrightarrow{R Z}$. Label the endpoint that leads away from the origin $C^{\prime}$.

4. Connect the rotated points.

The connected points $A^{\prime}, B^{\prime}$, and $C^{\prime}$ form the rotated figure.


