Guided Practice 4.2.2

Example 1

The Student Council wants to host a school-wide activity. Council members survey 40 students, asking them to choose either a field trip, a dance, or a talent show. The table below shows the survey results, with the surveyed students numbered 1–40. Construct a two-way frequency table to summarize the data.

Student	Grade	Activity	Student	Grade	Activity
1	10	FT	21	10	D
2	12	D	22	10	FT
3	10	TS	23	12	D
4	10	FT	24	11	D
5	11	D	25	11	TS
6	12	D	26	12	D
7	10	TS	27	12	D
8	10	FT	28	10	D
9	10	FT	29	11	D
10	11	TS	30	11	D
11	12	D	31	12	FT
12	10	TS	32	10	TS
13	11	TS	33	12	D
14	10	FT	34	11	D
15	11	D	35	11	FT
16	10	FT	36	11	FT
17	12	D	37	11	TS
18	10	FT	38	12	TS
19	12	D	39	11	FT
20	11	TS	40	12	TS

Key: TS = Talent show, FT = Field trip, D = Dance

1. Set up a tally table.

   There are two characteristics associated with each student: that student's grade and that student's choice of activity. Set up a table that shows "Grade" and "Activity choice" as categories, and all the different characteristics in each category.

Grade	Activity choice		
	Talent show	Field trip	Dance
10			
11			
12			

2. Tally the data.

   For each student, draw a tally mark that corresponds to that student's grade and choice of activity in the appropriate cell of the data table. The tally marks for students 1–5 are shown in the incomplete tally table below.

Grade	Activity choice		
	Talent show	Field trip	Dance
10	I	II	
11			I
12			I

   The complete tally table below shows the tally marks for all the students.

Grade	Activity choice		
	Talent show	Field trip	Dance
10	IIII	~~IIII~~ III	II
11	~~IIII~~	III	~~IIII~~ I
12	II	I	~~IIII~~ IIII

3. Create a two-way frequency table.

   Count the tally marks in each cell of your tally table. Then, create another table (a two-way frequency table) to show your count results. These results are frequencies. The completed two-way frequency table is shown below.

Grade	Activity choice		
	Talent show	**Field trip**	**Dance**
10	**4**	**8**	**2**
11	**5**	**3**	**6**
12	**2**	**1**	**9**

   Add all the frequencies; verify that their sum is 40 (since 40 students were surveyed).

   $4 + 8 + 2 + 5 + 3 + 6 + 2 + 1 + 9 = 40$

Example 2

The completed two-way frequency table from Example 1 is shown below. It shows the results of a survey designed to help the Student Council choose a school-wide activity.

Grade	Activity choice		
	Talent show	**Field trip**	**Dance**
10	4	8	2
11	5	3	6
12	2	1	9

Consider the following events that apply to a random student who participated in the survey.

TEN: The student is in the tenth grade.

TWELVE: The student is in the twelfth grade.

FT: The student prefers a field trip.

TS: The student prefers a talent show.

Compare $P(TEN|FT)$ and $P(FT|TEN)$. Are *TEN* and *FT* independent?

Compare $P(TWELVE|TS)$ and $P(TS|TWELVE)$. Are *TWELVE* and *TS* independent?

Interpret the results.

1. Find the totals of all the categories.

Grade	Activity choice			Total
	Talent show	Field trip	Dance	
10	4	8	2	14
11	5	3	6	14
12	2	1	9	12
Total	11	12	17	40

2. Compare $P(TEN|FT)$ and $P(FT|TEN)$.

$P(TEN|FT)=\frac{8}{12}\approx 0.667$ There were 12 votes for a field trip; 8 were by tenth graders.

$P(FT|TEN)=\frac{8}{14}\approx 0.571$ There were 14 votes by tenth graders; 8 were for a field trip.

0.667 > 0.571; therefore, $P(TEN|FT)>P(FT|TEN)$.

3. Determine if *TEN* and *FT* are independent.

Remember that events *A* and *B* are independent events if $P(B|A)=P(B)$ or if $P(A|B)=P(A)$.

Compare $P(TEN|FT)$ with $P(TEN)$ and $P(FT|TEN)$ with $P(FT)$.

$P(TEN\|FT)=\frac{8}{12}\approx 0.667$	There were 12 votes for a field trip; 8 were by tenth graders.
$P(TEN)=\frac{14}{40}=0.35$	There were 40 votes in all; 14 were by tenth graders.

$0.667 \neq 0.35$; therefore, $P(TEN|FT) \neq P(TEN)$.

$P(FT\|TEN)=\frac{8}{14}\approx 0.571$	There were 14 votes by tenth graders; 8 were for a field trip.
$P(FT)=\frac{12}{40}=0.3$	There were 40 votes in all; 12 were for a field trip.

$0.571 \neq 0.3$; therefore, $P(FT|TEN) \neq P(FT)$.

Based on the data, *TEN* and *FT* seem to be dependent because $P(TEN|FT) \neq P(TEN)$ and $P(FT|TEN) \neq P(FT)$.

4. Interpret the results for $P(TEN|FT)$ and $P(FT|TEN)$.

$P(TEN|FT)$ is the probability that a student is in the tenth grade given that he prefers a field trip.

$P(FT|TEN)$ is the probability that a student prefers a field trip given that he is in the tenth grade.

The fact that *TEN* and *FT* are dependent means that being in the tenth grade affects the probability that a student prefers a field trip, and preferring a field trip affects the probability that a student is in the tenth grade. In this case, being in the tenth grade increases the probability that a student prefers a field trip because $P(FT|TEN) > P(FT)$. Also, preferring a field trip increases the probability that a student is in the tenth grade because $P(TEN|FT) > P(TEN)$.

$P(TEN|FT) > P(FT|TEN)$ means that it is more likely that a student is in the tenth grade given that he prefers a field trip than it is that a student prefers a field trip given that he is in the tenth grade.

5. Compare $P(TWELVE|TS)$ and $P(TS|TWELVE)$.

$P(TWELVE|TS)=\frac{2}{11}\approx 0.182$ — There were 11 votes for a talent show; 2 were by twelfth graders.

$P(TS|TWELVE)=\frac{2}{12}\approx 0.167$ — There were 12 votes by twelfth graders; 2 were for a talent show.

0.182 > 0.167; therefore, $P(TWELVE|TS) > P(TS|TWELVE)$, but they are close in value. The values are approximately 18% and 17%.

6. Determine if *TWELVE* and *TS* are independent.

Events *A* and *B* are independent events if $P(B|A)=P(B)$ or if $P(A|B)=P(A)$.

Compare $P(TWELVE|TS)$ with $P(TWELVE)$ and $P(TS|TWELVE)$ with $P(TS)$.

$P(TWELVE|TS)=\frac{2}{11}\approx 0.182$ — There were 11 votes for a talent show; 2 were by twelfth graders.

$P(TWELVE)=\frac{12}{40}=0.3$ — There were 40 votes in all; 12 were by twelfth graders.

$0.182 \neq 0.3$; therefore, $P(TWELVE|TS) \neq P(TWELVE)$.

$P(TS|TWELVE)=\frac{2}{12}\approx 0.167$ — There were 12 votes by twelfth graders; 2 were for a talent show.

$P(TS)=\frac{11}{40}=0.275$ — There were 40 votes in all; 11 were for a talent show.

$0.167 \neq 0.275$; therefore, $P(TS|TWELVE) \neq P(TS)$.

Based on the data, *TWELVE* and *TS* are dependent because $P(TWELVE|TS) \neq P(TWELVE)$ and $P(TS|TWELVE) \neq P(TS)$.

7. Interpret the results for $P(TWELVE|TS)$ and $P(TS|TWELVE)$.

   $P(TWELVE|TS)$ is the probability that a student is in the twelfth grade given that the student prefers a talent show.

   $P(TS|TWELVE)$ is the probability that a student prefers a talent show given that the student is in the twelfth grade.

   The fact that *TWELVE* and *TS* are dependent means that being in the twelfth grade affects the probability that a student prefers a talent show, and preferring a talent show affects the probability that a student is in the twelfth grade. In this case, being in the twelfth grade decreases the probability that a student prefers a talent show because $P(TS|TWELVE) < P(TS)$. And preferring a talent show decreases the probability that a student is in the twelfth grade because $P(TWELVE|TS) < P(TWELVE)$.

   $P(TWELVE|TS) > P(TS|TWELVE)$, but they are close in value, differing by only about 1%. So it is almost equally likely that a student is in the twelfth grade given that the student prefers a talent show, as it is that a student prefers a talent show given that the student is in the twelfth grade.

Example 3

A cafeteria manager recorded the choices of 100 students who each chose one food item and one beverage. The table shows the data.

Beverage choice	Food choice	
	Burger	Pizza
Milk	34	26
Iced tea	24	16

Consider the following events that apply to a randomly chosen student who selects one food item and one beverage.

M: The student selects milk.

B: The student selects a burger.

P: The student selects pizza.

Compare $P(M|B)$ and $P(B|M)$. Are M and B independent?

Compare $P(M|P)$ and $P(P|M)$. Are M and P independent?

Interpret the results.

1. Find the totals of all the categories.

Beverage choice	Food choice		Total
	Burger	Pizza	
Milk	34	26	60
Iced tea	24	16	40
Total	58	42	100

2. Compare $P(M|B)$ and $P(B|M)$.

$P(M\|B)=\frac{34}{58}\approx 0.586$	58 burgers were chosen; 34 were accompanied by milk.
$P(B\|M)=\frac{34}{60}\approx 0.567$	60 servings of milk were chosen; 34 were accompanied by a burger.

0.586 > 0.567; therefore, $P(M|B) > P(B|M)$, but they are close in value. The values are approximately 59% and 57%.

3. Determine if M and B are independent.

Events A and B are independent if $P(B|A)=P(B)$ or if $P(A|B)=P(A)$.

Compare $P(M|B)$ with $P(M)$ and $P(B|M)$ with $P(B)$.

$P(M\|B)=\frac{34}{58}\approx 0.586$	58 burgers were chosen; 34 were accompanied by milk.
$P(M)=\frac{60}{100}=0.6$	There were 100 students, and 60 of them chose milk.

$0.586 \approx 0.6$; therefore, $P(M|B) \approx P(M)$.

$P(B\|M)=\frac{34}{60}\approx 0.567$	60 servings of milk were chosen; 34 were accompanied by a burger.
$P(B)=\frac{58}{100}=0.58$	There were 100 students, and 58 of them chose a burger.

$0.567 \approx 0.58$; therefore, $P(B|M) \approx P(B)$.

Based on the data, M and B seem to be independent because $P(M|B) \approx P(M)$ and $P(B|M) \approx P(B)$.

4. Interpret the results for $P(M|B)$ and $P(B|M)$.

   $P(M|B)$ represents the probability of a student choosing milk given that he chooses a burger.

   $P(B|M)$ represents the probability of a student choosing a burger given that he chooses milk.

   The conclusion that M and B are independent based on probabilities that are close in value means that choosing milk does not significantly affect the probability of choosing a burger, and choosing a burger does not significantly affect the probability of choosing milk.

   $P(M|B) > P(B|M)$, but they are close in value, differing by only about 1%. So it is almost equally likely that a student chooses milk given that the student chooses a burger, as it is that the student chooses a burger given that the student chooses milk. But because the choices are independent based on probabilities that are close in value, this can be restated more simply as follows: It is almost equally likely that a student chooses milk as that the student chooses a burger.

5. Compare $P(M|P)$ and $P(P|M)$.

   $P(M|P)=\frac{26}{42}\approx 0.619$ — 42 pizza servings were chosen; 26 were accompanied by milk.

   $P(P|M)=\frac{26}{60}\approx 0.433$ — 60 servings of milk were chosen; 26 were accompanied by pizza.

   0.619 > 0.433; therefore, $P(M|P) > P(P|M)$.

6. Determine if M and P are independent.

Events A and B are independent events if $P(B|A) = P(B)$ or if $P(A|B) = P(A)$.

Compare $P(M|P)$ with $P(M)$ and $P(P|M)$ with $P(P)$.

$P(M|P) = \frac{26}{42} \approx 0.619$ — 42 pizza servings were chosen; 26 were accompanied by milk.

$P(M) = \frac{60}{100} = 0.6$ — There were 100 students, and 60 of them chose milk.

$0.619 \approx 0.6$; therefore, $P(M|P) \approx P(M)$.

$P(P|M) = \frac{26}{60} \approx 0.433$ — 60 servings of milk were chosen; 26 were accompanied by pizza.

$P(P) = \frac{42}{100} = 0.42$ — There were 100 students, and 42 of them chose pizza.

$0.433 \approx 0.42$; therefore, $P(P|M) \approx P(P)$.

Based on the data, M and P seem to be independent because $P(M|P) \approx P(M)$ and $P(P|M) \approx P(P)$.

7. Interpret the results for $P(M|P)$ and $P(P|M)$.

$P(M|P)$ represents the probability that a student chooses milk given that the student chooses pizza.

$P(P|M)$ represents the probability that a student chooses pizza given that the student chooses milk.

The conclusion that M and P are independent based on probabilities that are close in value means that choosing milk does not significantly affect the probability of choosing pizza, and choosing pizza does not significantly affect the probability of choosing milk.

$P(M|P) > P(P|M)$ means it is more likely that a student chooses milk given that the student chooses pizza than it is that a student chooses pizza given that the student chooses milk. But since the choices are independent based on probabilities that are close in value, this can be restated as follows: It is more likely that a student chooses milk than that a student chooses pizza.