Guided Practice 4.1.2

Example 1

Bobbi tosses a coin 3 times. What is the probability that she gets exactly 2 heads? Write your answer as a fraction, as a decimal, and as a percent.

1. Identify the sample space and count the outcomes.

   Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

   There are 8 outcomes in the sample space.

2. Identify the outcomes in the event and count the outcomes.

   There are 3 outcomes in the event "exactly 2 heads:" HHT, HTH, and THH.

3. Apply the formula for the probability of an event.

   Divide the number of outcomes for the event Bobbi is hoping for (3) by the total number of possible outcomes (8).

   $$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$$

   $$P(\text{exactly 2 heads}) = \frac{3}{8} = 0.375 = 37.5\%$$

Example 2

Donte is playing a card game with a standard 52-card deck. He's hoping for a club or a face card on his first draw. What is the probability that he draws a club or a face card on his first draw?

1. Identify the sample space and count the outcomes.

   The sample space is the set of all cards in the deck, so there are 52 outcomes.

2. Identify the outcomes in the event and count the outcomes.

   You can use a table to show the sample space. Then identify and count the cards that are either a club or a face card or both a club and a face card.

Suit	2	3	4	5	6	7	8	9	10	J	Q	K	A
Spade										✔	✔	✔	
Club	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔	✔
Diamond										✔	✔	✔	
Heart										✔	✔	✔	

   The event "club or face card" has 22 outcomes.

3. Apply the formula for the probability of an event.

$$P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$$

$$P(\text{club or face card}) = \frac{22}{52} = \frac{11}{26} \approx 0.42$$

4. Apply the Addition Rule to verify your answer.

   Let A be the event "club." There are 13 clubs, so A has 13 outcomes.

   Let B be the event "face card." There are 12 face cards, so B has 12 outcomes.

   The event "A and B" is the event "club and face card," which has 3 outcomes: jack of clubs, queen of clubs, and king of clubs.

   Apply the Addition Rule.

$$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$$

$$P(\text{club or face card}) = P(\text{club}) + P(\text{face card}) - P(\text{club and face card})$$

$$P(\text{club or face card}) = \frac{13}{52} + \frac{12}{52} - \frac{3}{52} = \frac{22}{52} = \frac{11}{26} \approx 0.42$$

   The Addition Rule answer checks out with the probability found in step 3, so the probability of the event is approximately 0.42.

Example 3

Corrine is playing a board game. To find the number of spaces to move, she rolls a pair of dice. On her next roll she wants doubles or a sum of 10. What is the probability that she rolls doubles or a sum of 10 on her next roll? Interpret your answer in terms of a uniform probability model.

1. Identify the sample space and count the outcomes.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

There are 36 outcomes in the sample space.

2. Apply the Addition Rule to find the probability that she rolls doubles or a sum of 10 on her next roll.

Let A be the event "doubles." Event A has 6 outcomes: (1, 1), (2, 2), (3, 3), (4, 4), (5, 5), and (6, 6).

Let B be the event "sum of 10." Event B has 3 outcomes: (4, 6), (5, 5), and (6, 4).

The event "A and B" is the event "doubles and sum of 10," which has 1 outcome: (5, 5).

Apply the Addition Rule.

$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

$P(\text{doubles or sum of } 10) = P(\text{doubles}) + P(\text{sum of } 10) - P(\text{doubles and sum of } 10)$

$$P(\text{doubles or sum of } 10) = \frac{6}{36} + \frac{3}{36} - \frac{1}{36} = \frac{8}{36} = \frac{2}{9} \approx 0.22 \approx 22\%$$

3. Interpret the answer in terms of a uniform probability model.

   The probabilities used in the application of the Addition Rule, $\frac{6}{36}$, $\frac{3}{36}$, and $\frac{1}{36}$, are found by using the formula $P(E) = \frac{\text{number of outcomes in } E}{\text{number of outcomes in the sample space}}$.

   This formula is a uniform probability model, which requires that all outcomes in the sample space be equally likely. It is reasonable to assume that the dice Corrine rolls are fair, so that all outcomes in the sample space are equally likely. Therefore, the answer is valid and can serve as a reasonable predictor. You can predict that the relative frequency of getting doubles or a sum of 10 will be about 22% for a large number of dice rolls. That is, you can predict that Corrine will get doubles or a sum of 10 about 22% of the time if she rolls the dice a large number of times.

Example 4

Students at Rolling Hills High School receive an achievement award for either performing community service or making the honor roll. The school has 500 students and 180 of them received the award. There were 125 students who performed community service and 75 students who made the honor roll. What is the probability that a randomly chosen student at Rolling Hills High School performed community service and made the honor roll?

1. Define the sample space and state its number of outcomes.

   The sample space is the set of all students at the school; it has 500 outcomes.

2. Define events that are associated with the numbers in the problem and state their probabilities.

   Let A be the event "performed community service." Then $P(A)=\frac{125}{500}$.

   Let B be the event "made the honor roll." Then $P(B)=\frac{75}{500}$.

   The event "A or B" is the event "performed community service or made the honor roll," and can also be written $A\cup B$. $P(A\cup B)=\frac{180}{500}$ because 180 students received the award for either community service or making the honor roll.

3. Write the Addition Rule and solve it for $P(A$ and $B)$, which is the probability of the event "performed community service and made the honor roll." $P(A$ and $B)$ can also be written $P\left(A\cap B\right)$.

   $P(A\cup B)=P(A)+P(B)-P(A\cap B)$

$\frac{180}{500}=\frac{125}{500}+\frac{75}{500}-P(A\cap B)$	Substitute the known probabilities.
$\frac{180}{500}=\frac{200}{500}-P(A\cap B)$	Simplify.
$-\frac{20}{500}=-P(A\cap B)$	Subtract $\frac{200}{500}$ from both sides.
$\frac{20}{500}=P(A\cap B)$	Multiply both sides by –1.

   The probability that a randomly chosen student at Rolling Hills High School has performed community service and is on the honor roll is $\frac{20}{500}=\frac{1}{25}$, or 4%.