Guided Practice 4.1.1

Example 1

The table that follows shows a group of students and the extra-curricular activities in which they participate.

Student	Chess club	Debate club	Band	German club
Asher			✔	
Ray				
Eva		✔		✔
Merida			✔	
Ysabel		✔	✔	
Ben	✔			
Nate	✔			✔

A student is chosen from the group at random. List the sample space and then describe each of the following events.

{Eva, Nate} {Ysabel} {Ben, Nate, Eva} {Ray, Eva, Ben, Nate}

1. The sample space is the set of all possible outcomes.

   The sample space is {Asher, Ray, Eva, Merida, Ysabel, Ben, Nate}.

2. Describe the event {Eva, Nate}.

   Eva is in the debate club and the German club.

   Nate is in the chess club and the German club.

   To write a description of {Eva, Nate}, consider using the word "or." The activities mentioned are: debate club, German club, and chess club. You might consider this description: "in the debate club, German club, or chess club." But that description would also include Ysabel, who is in the debate club, and Ben, who is in the chess club. So that description is not correct.

   Consider using the word "and." Identify the activity or activities Eva and Nate have in common. They are both in the German club. There are no other students in the German club, so the event can be described as "in the German club."

   A correct description is "the chosen student is in the German club."

3. Describe the event {Ysabel}.

   Ysabel is in the debate club and the band, and there are no other students in both of those activities.

   Therefore, a correct description is "the chosen student is in the debate club and the band."

4. Describe the event {Ben, Nate, Eva}.

   Ben is in the chess club.

   Nate is in the chess club and the German club.

   Eva is in the debate club and the German club.

   There is no activity common to all three students, so the description will not use the word "and."

   Consider using the word "or." The activities mentioned are: chess club, German club, and debate club. You might consider this description: "in the chess club, German club, or debate club." But that description would also include Ysabel, who is in the debate club. So that description is not correct.

   A correct description is "the chosen student is in the chess club or German club."

5. Describe the event {Ray, Eva, Ben, Nate}.

   Ray is in no activity.

   Eva is in the debate club and the German club.

   Ben is in the chess club.

   Nate is in the chess club and the German club.

   Since Ray is in no activity, consider using the word "not." Identify any activity that none of the four students are in.

   A correct description is "the chosen student is not in the band."

Instruction

Example 2

Some students were asked what pets they have at home. The following table shows the results of the survey, with the students identified by numbers.

Student	Dog	Cat	Hamster	Bird	Fish
1					
2	✔	✔			
3					
4	✔				
5			✔		✔
6					✔
7	✔	✔		✔	
8					
9		✔		✔	
10	✔	✔			

A student is chosen from the group at random. Consider the following events.

D: The student has a dog.

C: The student has a cat.

H: The student has a hamster.

B: The student has a bird.

F: The student has a fish.

Describe each of the following events by listing outcomes.

C $D \cap B$ $H \cup F$ $\overline{B}$ $\overline{D \cap C}$ $\overline{D \cup C}$

1. List the outcomes of *C*.

   Identify the students who have a cat. List those student numbers in set notation.

   $C = \{2, 7, 9, 10\}$

2. List the outcomes of $D \cap B$.

   Identify the students who have a dog. List those student numbers in set notation.

   $D = \{2, 4, 7, 10\}$

   Identify the students who have a bird. List those student numbers in set notation.

   $B = \{7, 9\}$

   $D \cap B$ is the intersection of events D and B or the student(s) who have both a dog and a bird. Identify the outcome(s) common to both events.

   $D \cap B = \{7\}$

3. List the outcomes of $H \cup F$.

   Identify the students who have a hamster. List those student numbers in set notation.

   $H = \{5\}$

   Identify the students who have a fish. List those student numbers in set notation.

   $F = \{5, 6\}$

   $H \cup F$ is the union of events H and F or the students who have a hamster or a fish or who have both. Identify the outcomes that appear in either event or both events.

   $H \cup F = \{5, 6\}$

4. List the outcomes of $\overline{B}$.

   Identify the students who have a bird. List those student numbers in set notation.

   $B = \{7, 9\}$

   Sample space = {1, 2, 3, 4, 5, 6, 7, 8, 9, 10}

   $\overline{B}$ is the set of all outcomes that are in the sample space, but not in B. In other words, $\overline{B}$ is all the students who don't have a bird.

   $\overline{B} = \{1, 2, 3, 4, 5, 6, 8, 10\}$

5. List the outcomes of $\overline{D \cap C}$.

   To find the outcomes of $\overline{D \cap C}$, we must first determine the outcomes of $D \cap C$.

   From step 2, we determined the outcomes in D to be as follows:

   $D = \{2, 4, 7, 10\}$

   From step 1, we determined the outcomes in C to be as follows:

   $C = \{2, 7, 9, 10\}$

   The intersection of D and C, then, is the outcome(s) that are in both D and C.

   $D \cap C = \{2, 7, 10\}$

   $\overline{D \cap C}$ is the set of all outcomes that are in the sample space, but not in $D \cap C$.

   $\overline{D \cap C} = \{1, 3, 4, 5, 6, 8, 9\}$

6. List the outcomes of $\overline{D \cup C}$.

   First, determine the outcomes of $D \cup C$.

   $D = \{2, 4, 7, 10\}$

   $C = \{2, 7, 9, 10\}$

   $D \cup C = \{2, 4, 7, 9, 10\}$

   $\overline{D \cup C}$ is the set of all outcomes that are in the sample space, but not in $D \cup C$.

   $\overline{D \cup C} = \{1, 3, 5, 6, 8\}$

Example 3

Hector has entered the following names in the contact list of his new cell phone: Alicia, Brisa, Steve, Don, and Ellis. He chooses one of the names at random to call. Consider the following events.

B: The name begins with a vowel.

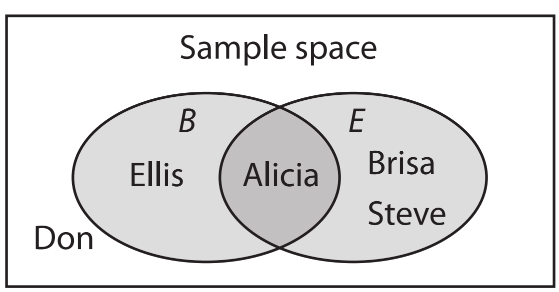
E: The name ends with a vowel.

Draw a Venn diagram to show the sample space and the events *B* and *E*. Then describe each of the following events by listing outcomes.

B $\quad E$ $\quad B \cap E$ $\quad B \cup E$ $\quad \overline{B}$ $\quad \overline{B \cup E}$

1. Draw a Venn diagram. Use a rectangle for the sample space. Use circles or elliptical shapes for the events *B* and *E*.

   Write the students' names in the appropriate sections to show what events they are in.



2. List the outcomes of B.

   B = {Ellis, Alicia}

3. List the outcomes of E.

   E = {Alicia, Brisa, Steve}

4. List the outcomes of $B \cap E$.

   B = {Ellis, Alicia}

   E = {Alicia, Brisa, Steve}

   $B \cap E$ is the intersection of events B and E. Identify the outcome(s) common to both events.

   $B \cap E$ = {Alicia}

5. List the outcomes of $B \cup E$.

   B = {Ellis, Alicia}

   E = {Alicia, Brisa, Steve}

   $B \cup E$ is the union of events B and E. Identify the outcomes that appear in either event or both events.

   $B \cup E$ = {Ellis, Alicia, Brisa, Steve}

6. List the outcomes of $\overline{B}$.

   B = {Ellis, Alicia}

   Sample space = {Alicia, Brisa, Steve, Don, Ellis}

   $\overline{B}$ is the set of all outcomes that are in the sample space, but not in B.

   $\overline{B}$ = {Brisa, Steve, Don}

7. List the outcomes of $\overline{B \cup E}$.

B = {Ellis, Alicia}

E = {Alicia, Brisa, Steve}

$B \cup E$ = {Ellis, Alicia, Brisa, Steve}

Sample space = {Alicia, Brisa, Steve, Don, Ellis}

$\overline{B \cup E}$ is the set of all outcomes that are in the sample space, but not in $B \cup E$.

$\overline{B \cup E}$ = {Don}

Example 4

An experiment consists of tossing a coin three times. Consider the following events.

A: The first toss is heads.

B: The second toss is heads.

C: There are two consecutive heads.

D: There are two consecutive tails.

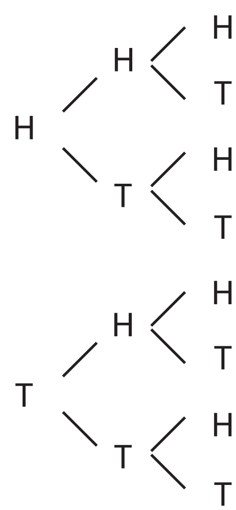
E: The first toss is heads and the second toss is heads.

F: There are neither two consecutive heads nor two consecutive tails.

List the sample space. Then express events *E* and *F* in terms of other events and list the outcomes of *E* and *F*.

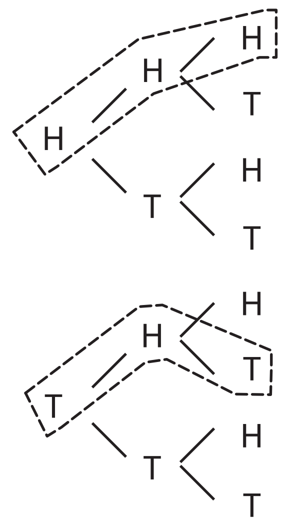
1. The sample space is the set of all possible outcomes.

   The tree diagram below shows all possible outcomes, using H for heads and T for tails.



(*continued*)

To identify all the possible outcomes, trace all the different paths from left to right. The following diagram shows two different paths. The first indicated path identifies the outcome HHH, which means heads on all three coin tosses. The second indicated path identifies the outcome THT, which means tails, then heads, then tails.



There are eight different paths, indicating the following sample space:

{HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

2. To express event *E* in terms of other events, first note how the descriptions of *A*, *B*, and *E* are related.

   A: The first toss is heads.

   B: The second toss is heads.

   E: The first toss is heads and the second toss is heads.

   Using the word "and" in this way indicates that *E* is the intersection of *A* and *B*.

   $E = A \cap B$

   To list the outcomes of *E*, list all the outcomes that are in both *A* and *B*.

   $E = A \cap B = \{HHH, HHT\}$

3. To express event F in terms of other events, first note how the descriptions of C, D, and F are related.

   C: There are two consecutive heads.

   D: There are two consecutive tails.

   F: There are neither two consecutive heads nor two consecutive tails.

   The union of C and D, denoted $C \cup D$, is the event that there are either two consecutive heads or two consecutive tails.

   $C \cup D$ = {HHH, HHT, HTT, THH, TTH, TTT}

   F is the event that there are *neither* two consecutive heads *nor* two consecutive tails. So F is the complement of $C \cup D$.

   $F = \overline{C \cup D}$

   To list the outcomes of F, list all the outcomes that are in the sample space, but not in $C \cup D$.

   Sample space = {HHH, HHT, HTH, HTT, THH, THT, TTH, TTT}

   $F = \overline{C \cup D}$ = {HTH, THT}

Example 5

An experiment consists of rolling a pair of dice. How many ways can you roll the dice so that the product of the two numbers rolled is less than their sum?

1. Begin by showing the sample space.

   This diagram of ordered pairs shows the sample space.

(1, 1)	(1, 2)	(1, 3)	(1, 4)	(1, 5)	(1, 6)
(2, 1)	(2, 2)	(2, 3)	(2, 4)	(2, 5)	(2, 6)
(3, 1)	(3, 2)	(3, 3)	(3, 4)	(3, 5)	(3, 6)
(4, 1)	(4, 2)	(4, 3)	(4, 4)	(4, 5)	(4, 6)
(5, 1)	(5, 2)	(5, 3)	(5, 4)	(5, 5)	(5, 6)
(6, 1)	(6, 2)	(6, 3)	(6, 4)	(6, 5)	(6, 6)

   Key: (2, 3) means 2 on the first die and 3 on the second die.

2. Identify all the outcomes in the event "the product is less than the sum."

For (1, 1), the product is $1 \times 1 = 1$ and the sum is $1 + 1 = 2$, so the product is less than the sum.

For (1, 2), the product is $1 \times 2 = 2$ and the sum is $1 + 2 = 3$, so the product is less than the sum.

This table shows all the possible products and sums.

Outcome	Product	Sum	Product < sum	Outcome	Product	Sum	Product < sum
(1, 1)	1	2	Yes	(4, 1)	4	5	Yes
(1, 2)	2	3	Yes	(4, 2)	8	6	No
(1, 3)	3	4	Yes	(4, 3)	12	7	No
(1, 4)	4	5	Yes	(4, 4)	16	8	No
(1, 5)	5	6	Yes	(4, 5)	20	9	No
(1, 6)	6	7	Yes	(4, 6)	24	10	No
(2, 1)	2	3	Yes	(5, 1)	5	6	Yes
(2, 2)	4	4	No	(5, 2)	10	7	No
(2, 3)	6	5	No	(5, 3)	15	8	No
(2, 4)	8	6	No	(5, 4)	20	9	No
(2, 5)	10	7	No	(5, 5)	25	10	No
(2, 6)	12	8	No	(5, 6)	30	11	No
(3, 1)	3	4	Yes	(6, 1)	6	7	Yes
(3, 2)	6	5	No	(6, 2)	12	8	No
(3, 3)	9	6	No	(6, 3)	18	9	No
(3, 4)	12	7	No	(6, 4)	24	10	No
(3, 5)	15	8	No	(6, 5)	30	11	No
(3, 6)	18	9	No	(6, 6)	36	12	No

By checking all the outcomes in the sample space, you can verify that the product is less than the sum for only these outcomes:

(1, 1) (1, 2) (1, 3) (1, 4) (1, 5) (1, 6)
(2, 1) (3, 1) (4, 1) (5, 1) (6, 1)

3. Count the outcomes that meet the event criteria.

There are 11 ways to roll two dice so that the product is less than the sum.