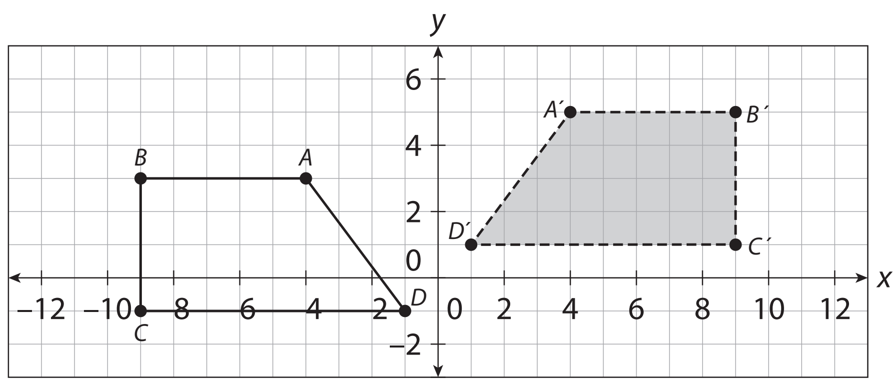
Guided Practice 4.1

Example 1

Consider the transformation of trapezoid $ABCD$ onto trapezoid $A'B'C'D'$. The coordinates for $ABCD$ are $A(-4, 3)$, $B(-9, 3)$, $C(-9, -1)$, and $D(-1, -1)$, and the coordinates for $A'B'C'D'$ are $A'(4, 5)$, $B'(9, 5)$, $C'(9, 1)$, and $D'(1, 1)$. Determine and perform a sequence of rigid motions that transformed $ABCD$ onto $A'B'C'D'$.

1. Graph each of the trapezoids on a coordinate plane.



2. Examine the resulting graph to determine whether any reflections or translations occurred.

   When the trapezoids are drawn, we can see that $A'B'C'D'$ is a reflection through the y-axis. We can also see that $A'B'C'D'$ moved up 2 units. This means a translation occurred.

3. What reflections could be used to transform the preimage *ABCD* to image $A'\ B'\ C'\ D'$?

   We determined a reflection occurred through the *y*-axis. To find the coordinates for a reflection over the *y*-axis, keep the *y*-coordinates the same and change the *x*-coordinates to the opposite.

   $r(x, y) = (-x, y)$

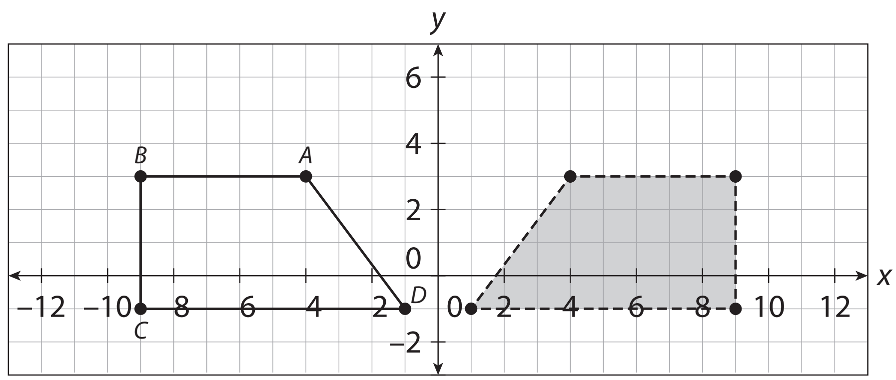
   $r(A(-4, 3)) = (4, 3)$

   $r(B(-9, 3)) = (9, 3)$

   $r(C(-9, -1)) = (9, -1)$

   $r(D(-1, -1)) = (1, -1)$

   We can now use the new coordinates to graph the reflected shape:



4. What translations could be used to transform the preimage $ABCD$ to image $A'\ B'\ C'\ D'$?

We determined that a translation occurred. To find the coordinates for a translation, add the horizontal change to the x-coordinate and the vertical change to the y-coordinate. In this case, there was no horizontal change, but the trapezoid was translated up 2 units. We will use the coordinates of the reflected shape we found in the previous step.

$T(x, y) = (x + a, y + b)$

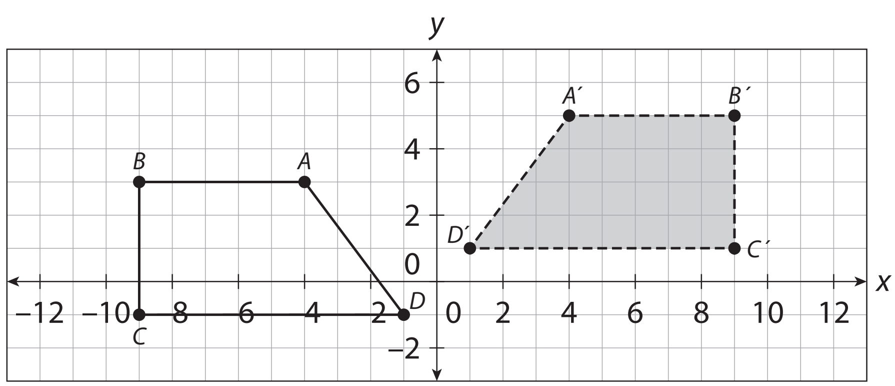
$T(4, 3) = (4 + 0, 3 + 2) = A'\ (4, 5)$

$T(9, 3) = (9 + 0, 3 + 2) = B'\ (9, 5)$

$T(9, -1) = (9 + 0, -1 + 2) = C'\ (9, 1)$

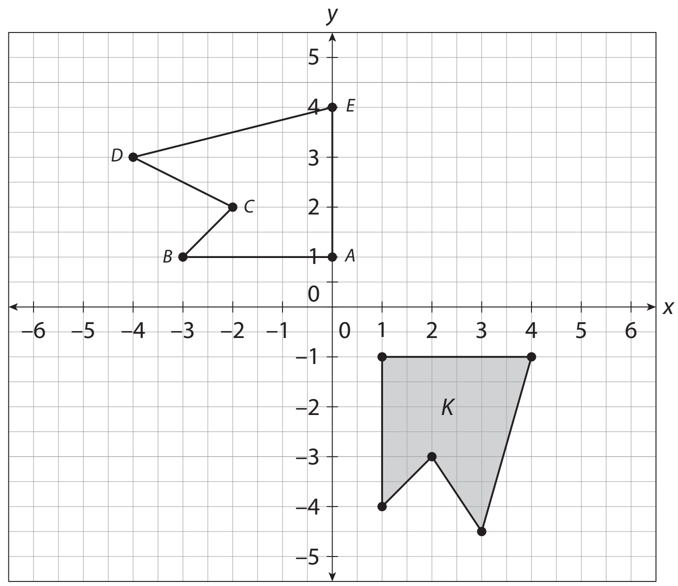
$T(1, -1) = (1 + 0, -1 + 2) = D'\ (1, 1)$

We can now use the new coordinates to graph the reflected and translated shape:



Example 2

Determine and perform a sequence of rigid motions that will transform figure *ABCDE* onto figure *K* with coordinates A' B' C' D' E' . Then determine whether the two figures are congruent.



1. Examine the graph to determine what sequence of rigid motions will transform figure *ABCDE* onto figure *K*.

   We can see that figure *K* is a 90° clockwise rotation of figure *ABCDE*. We can also see that both a reflection and a translation occurred.

2. Perform the first transformation by finding and graphing the coordinates.

   The first transformation is a 90° clockwise rotation of figure *ABCDE*. To perform this transformation, change the sign of the *x*-coordinates to the opposite and switch the order of the *x*- and *y*-coordinates.

   $R(x, y) = (y, -x)$

   $R(A(0, 1)) = (1, 0)$

   $R(B(-3, 1)) = (1, 3)$

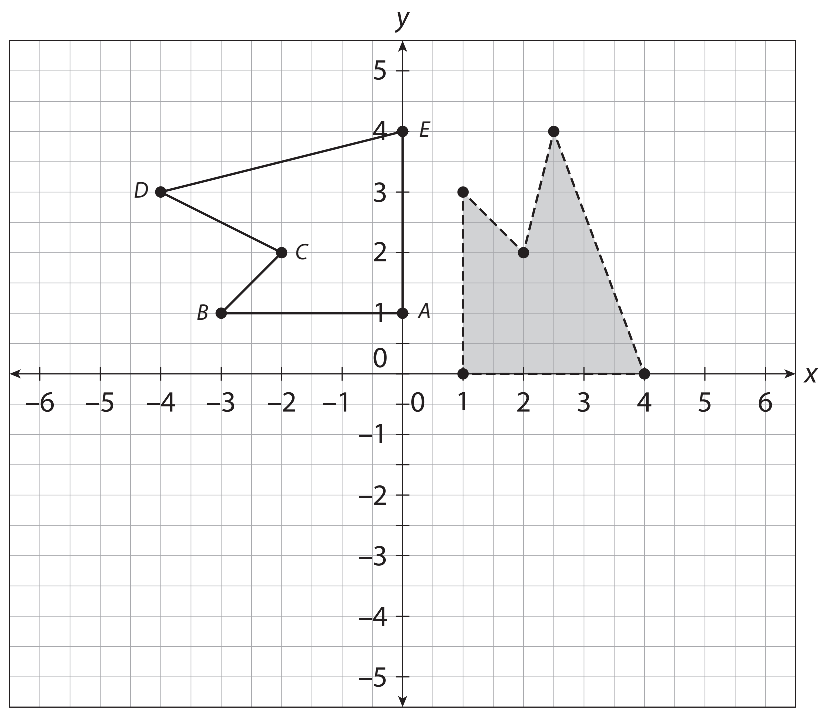
   $R(C(-2, 2)) = (2, 2)$

   $R(D(-4, 3)) = (3, 4)$

   $R(E(0, 4)) = (4, 0)$

(*continued*)

We can now use the new coordinates to graph the rotated shape:



3. Perform the second transformation by finding and graphing the coordinates.

The second transformation is a reflection through the *x*-axis. To perform this transformation, the *x*-coordinate remains the same and the *y*-coordinate changes to the opposite. We will use the coordinates we found in the previous step.

$r(x, y) = (x, -y)$

$r(A'\ (1, 0)) = (1, 0)$

$r(B'\ (1, 3)) = (1, -3)$

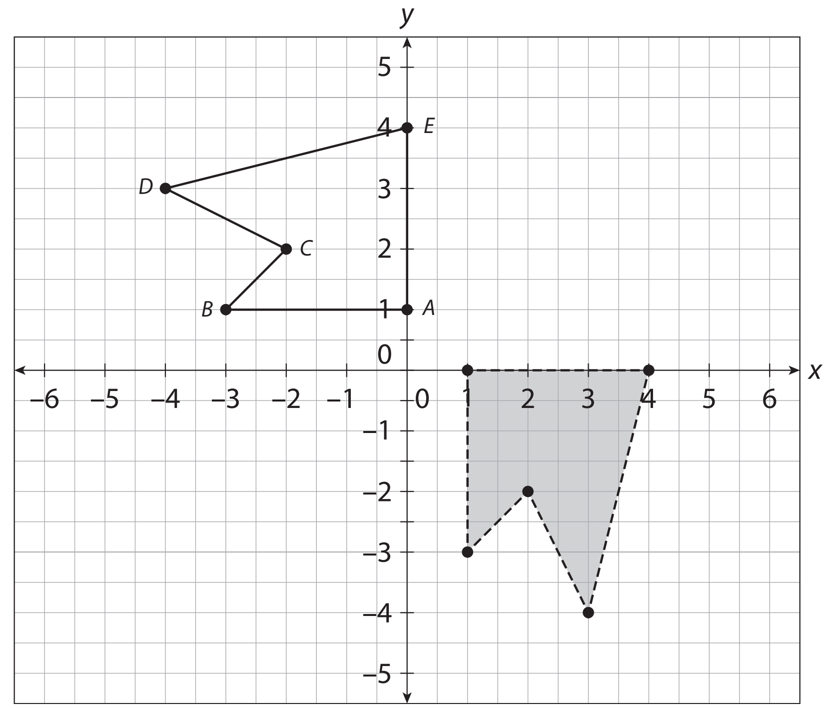
$r(C'\ (2, 2)) = (2, -2)$

$r(D'\ (3, 4)) = (3, -4)$

$r(E'\ (4, 0)) = (4, 0)$

(*continued*)

We can now use the new coordinates to graph the rotated and reflected shape:



4. Perform the third transformation by finding and graphing the coordinates.

   The third transformation is a translation. To perform this transformation, add the horizontal change to the x-coordinate and the vertical change to the y-coordinate. We need to translate the figure down by 1 unit, with no horizontal change.

   $T(x, y) = (x + a, y + b)$

   $T(A'\ (1, 0)) = (1 + 0, 0 - 1) = (1, -1)$

   $T(B'\ (1, -3)) = (1 + 0, -3 - 1) = (1, -4)$

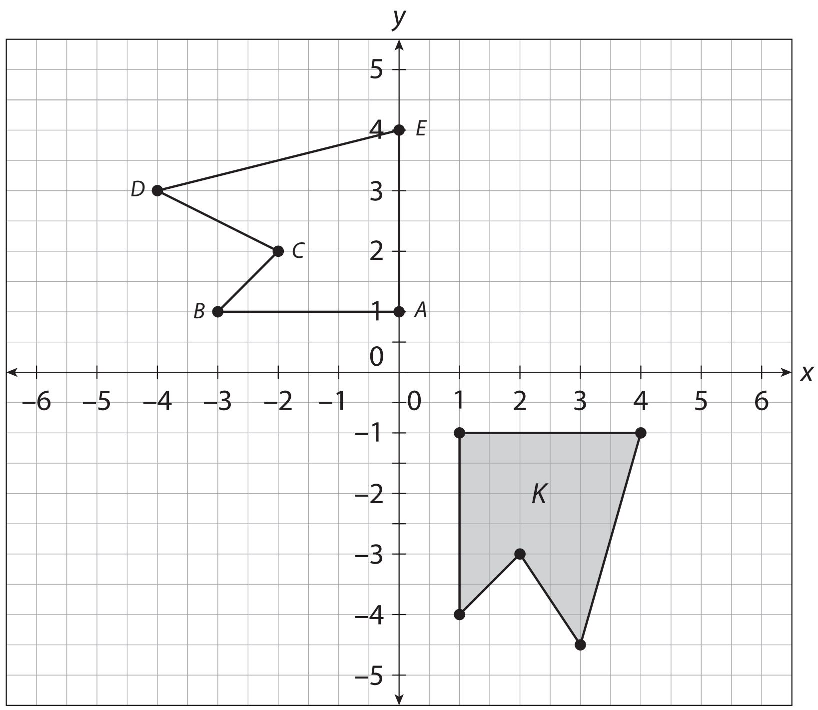
   $T(C'\ (2, -2)) = (2 + 0, -2 - 1) = (2, -3)$

   $T(D'\ (3, -4)) = (3 + 0, -4 - 1) = (3, -5)$

   $T(E'\ (4, 0)) = (4 + 0, 0 - 1) = (4, -1)$

(*continued*)

We can now use the new coordinates to graph the rotated, reflected, and translated shape:



5. Determine whether figure *ABCDE* and figure *K* are congruent.

At first glance, it appears the figures are congruent. We can confirm this by comparing the lengths of the corresponding sides. You can use the distance formula if needed: $d=\sqrt{(x_2-x_1)^2+(y_2-y_1)^2}$.

$\overline{AB}$ and its corresponding side on figure *K* are 3 units.

$\overline{BC}$ and its corresponding side on figure *K* are $\sqrt{2}$ units.

$\overline{CD}$ and its corresponding side on figure *K* are $\sqrt{5}$ units.

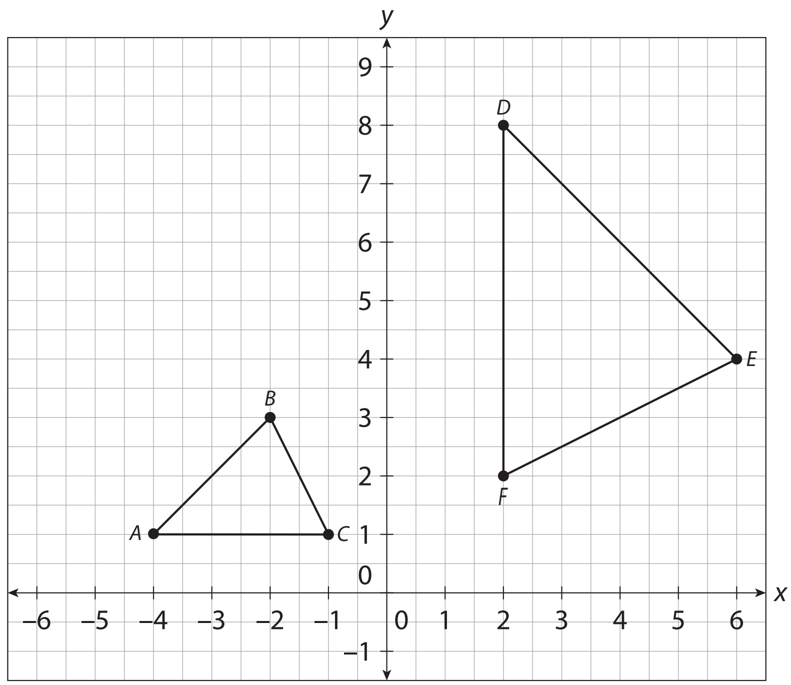
$\overline{DE}$ and its corresponding side on figure *K* are $\sqrt{17}$ units.

$\overline{EA}$ and its corresponding side on figure *K* are 3 units.

The corresponding parts of all the sides are congruent, so the figures are congruent.

Example 3

Determine and perform a sequence of transformations for ΔABC onto ΔDEF. Then determine whether the two figures are congruent.



1. Examine the graph to determine the sequence of transformations for ΔABC onto ΔDEF.

   We can see that ΔDEF is a 90° clockwise rotation of ΔABC. We can also see that ΔDEF is larger than ΔABC, so we will have to perform a dilation.

2. Perform the first transformation and find the new coordinates.

   The first transformation is a 90° clockwise rotation of ΔABC. To perform this transformation, change the x-coordinates to the opposite and switch the order of the x- and y-coordinates.

   $R(x, y) = (y, -x)$

   $R(A(-4, 1)) = (1, 4)$

   $R(B(-2, 3)) = (3, 2)$

   $R(C(-1, 1)) = (1, 1)$

3. Perform the second transformation and find the new coordinates.

The second transformation is a dilation. A dilation is performed by multiplying each coordinate by a common factor. By comparing the coordinates of the rotated ΔABC to the coordinates of ΔDEF, we can determine the scale factor of this dilation.

Coordinates of rotated ΔABC	A' (1, 4)	B' (3, 2)	C' (1, 1)
Coordinates of ΔDEF	D(2, 8)	E(6, 4)	F(2, 2)

We can see that from ΔABC to ΔDEF, each number in the ordered pair is multiplied by a scale factor of 2. To confirm this, use the formula for dilations.

$D(x, y) = (kx, ky)$

$D(A(1, 4)) = (2, 8)$

$D(B(3, 2)) = (6, 4)$

$D(C(1, 1)) = (2, 2)$

The coordinates for the rotated and dilated ΔABC will therefore be (2, 8), (6, 4), and (2, 2).

4. Determine whether ΔABC is congruent with ΔDEF.

The triangles are not congruent because they do not coincide. However, they are similar because their corresponding sides are proportional.