Guided Practice 2.6

Example 1

Given the function $f(x) = x^2$, identify the key features of the graph: the extremum, vertex, and y-intercept. Then sketch the graph.

1. Determine the extremum of the graph.

   The extreme value is a minimum when $a > 0$. It is a maximum when $a < 0$.

   Because $a = 1$, the graph opens upward and the quadratic has a minimum.

2. Determine the vertex of the graph.

   The minimum value occurs at the vertex.

   The vertex is of the form $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

   Use the original function $f(x) = x^2$ to find the values of a and b in order to find the x-coordinate of the vertex.

$x=\frac{-b}{2a}$	Formula to find the x-coordinate of the vertex of a parabola
$x=\frac{-(0)}{2(1)}$	Substitute 1 for a and 0 for b.
$x = 0$	Simplify.

   The x-coordinate of the vertex is 0.

   Substitute 0 into the original equation to find the y-coordinate.

$f(x) = x^2$	Original equation
$f(0) = (0)^2$	Substitute 0 for x.
$f(0) = 0$	Simplify.

   The y-coordinate of the vertex is 0.

   The vertex is located at (0, 0).

3. Determine the y-intercept of the graph.

   The y-intercept occurs when $x = 0$.

   The y-intercept of the function $f(x) = x^2$ is the same as the vertex, (0, 0).

   When the equation is written in standard form, the y-intercept is c.

4. Graph the function.

   Create a table of values and axis of symmetry to identify points on the graph.

   The axis of symmetry goes through the vertex, so the axis of symmetry is $x = 0$.

   For each point to the left of the axis of symmetry, there is another point the same distance on the right side of the axis and vice versa.

   Choose at least two values of x that are to the right and left of 0.

   Let's start with $x = 2$.

$f(x) = x^2$	Original equation
$f(2) = (2)^2$	Substitute 2 for x.
$f(2) = 4$	Simplify.

   An additional point is (2, 4).

   (2, 4) is 2 units to the right of the vertex. The point (–2, 4) is 2 units to the left of the vertex, so (–2, 4) is also on the graph.

   To find another set of points on the graph, let's evaluate the original equation for $x = 3$.

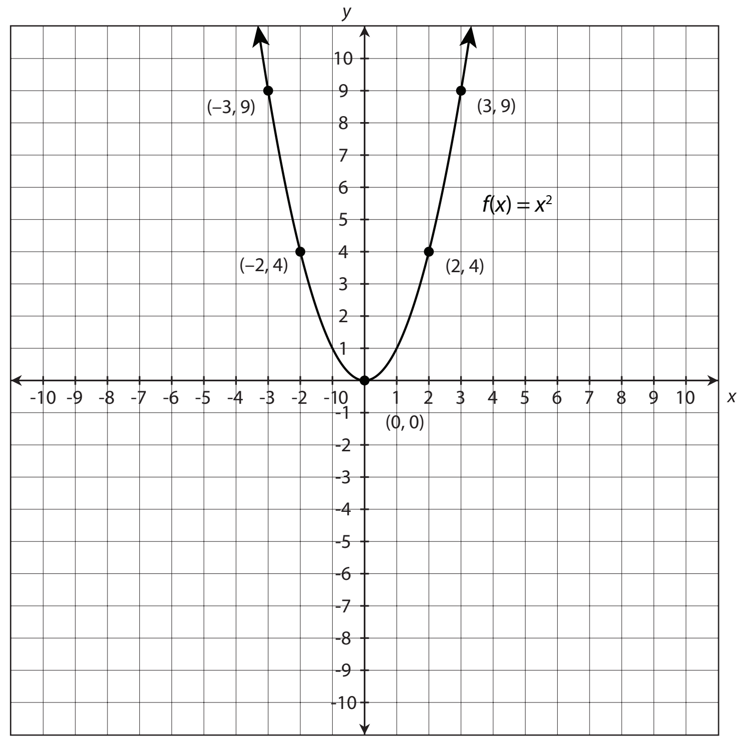
$f(x) = x^2$	Original equation
$f(3) = (3)^2$	Substitute 3 for x.
$f(3) = 9$	Simplify.

   An additional point is (3, 9).

   (3, 9) is 3 units to the right of the vertex. The point (–3, 9) is 3 units to the left of the vertex, so (–3, 9) is also on the graph.

(*continued*)

Plot the points and join them with a smooth curve.



Example 2

Given the function $f(x) = -2x^2 + 16x - 30$, identify the key features of the graph: the extremum, vertex, and y-intercept. Then sketch the graph.

1. Determine the extremum of the graph.

   The extreme value is a minimum when $a > 0$. It is a maximum when $a < 0$.

   Because $a = -2$, the graph opens downward and the quadratic has a maximum.

2. Determine the vertex of the graph.

   The maximum value occurs at the vertex.

   The vertex is of the form $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

   Use the original equation $f(x) = -2x^2 + 16x - 30$ to find the values of a and b in order to find the x-coordinate of the vertex.

$x=\frac{-b}{2a}$	Formula to find the x-coordinate of the vertex of a parabola
$x=\frac{-(16)}{2(-2)}$	Substitute –2 for a and 16 for b.
$x = 4$	Simplify.

   The x-coordinate of the vertex is 4.

   Substitute 4 into the original equation to find the y-coordinate.

$f(x) = -2x^2 + 16x - 30$	Original equation
$f(4) = -2(4)^2 + 16(4) - 30$	Substitute 4 for x.
$f(4) = 2$	Simplify.

   The y-coordinate of the vertex is 2.

   The vertex is located at (4, 2).

3. Determine the y-intercept of the graph.

   The y-intercept occurs when $x = 0$.

   Substitute 0 for x in the original equation.

$f(x) = -2x^2 + 16x - 30$	Original equation
$f(0) = -2(0)^2 + 16(0) - 30$	Substitute 0 for x.
$f(0) = -30$	Simplify.

   The y-intercept is (0, –30).

   When the quadratic equation is written in standard form, the y-intercept is c.

4. Graph the function.

Use symmetry to identify additional points on the graph.

The axis of symmetry goes through the vertex, so the axis of symmetry is $x = 4$.

For each point to the left of the axis of symmetry, there is another point the same distance on the right side of the axis and vice versa.

The point (0, –30) is on the graph, and 0 is 4 units to the left of the axis of symmetry.

The point that is 4 units to the right of the axis is 8, so the point (8, –30) is also on the graph.

Determine two additional points on the graph.

Choose an x-value to the left or right of the vertex and find the corresponding y-value.

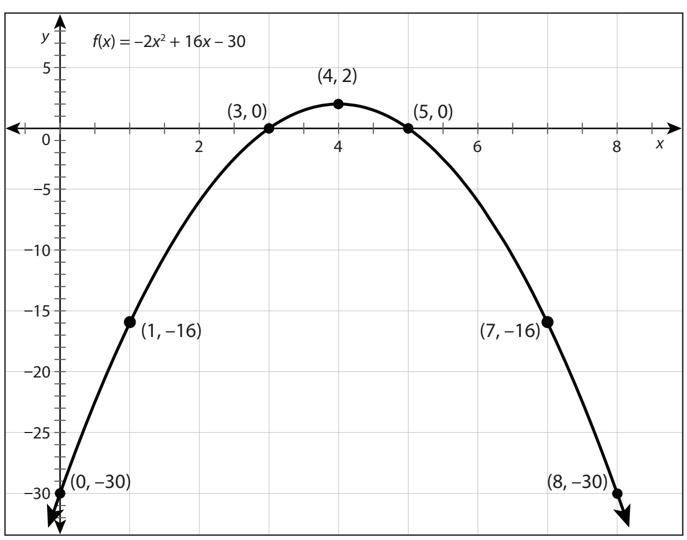
$f(x) = -2x^2 + 16x - 30$	Original equation
$f(1) = -2(1)^2 + 16(1) - 30$	Substitute 1 for x.
$f(1) = -16$	Simplify.

An additional point is (1, –16).

(1, –16) is 3 units to the left of the axis of symmetry.

The point that is 3 units to the right of the axis is 7, so the point (7, –16) is also on the graph.

Plot the points and join them with a smooth curve.



Example 3

Given the function $f(x) = x^2 + 6x + 9$, identify the key features of its graph: the extremum, vertex, and y-intercept. Then sketch the graph.

1. Determine the extremum of the graph.

   The extreme value is a minimum when $a > 0$. It is a maximum when $a < 0$.

   Because $a = 1$, the graph opens upward and the quadratic has a minimum.

2. Determine the vertex of the graph.

   The minimum value occurs at the vertex.

   The vertex is of the form $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

   Use the original function $f(x) = x^2 + 6x + 9$ to find the values of a and b in order to find the x-coordinate of the vertex.

$x = \frac{-b}{2a}$	Formula to find the x-coordinate of the vertex of a parabola
$x = \frac{-(6)}{2(1)}$	Substitute 1 for a and 6 for b.
$x = -3$	Simplify.

   The x-coordinate of the vertex is –3.

   Substitute –3 into the original equation to find the y-coordinate.

$f(x) = x^2 + 6x + 9$	Original equation
$f(-3) = (-3)^2 + 6(-3) + 9$	Substitute –3 for x.
$f(-3) = 0$	Simplify.

   The y-coordinate of the vertex is 0.

   The vertex is located at (–3, 0).

3. Determine the y-intercept of the graph.

The y-intercept occurs when $x = 0$.

Substitute 0 for x in the original equation.

$f(x) = x^2 + 6x + 9$	Original equation
$f(0) = (0)^2 + 6(0) + 9$	Substitute 0 for x.
$f(0) = 9$	Simplify.

The y-intercept is (0, 9).

4. Graph the function.

Use symmetry to identify an additional point on the graph.

The axis of symmetry goes through the vertex, so the axis of symmetry is $x = -3$.

For each point to the left of the axis of symmetry, there is another point the same distance on the right side of the axis and vice versa.

The point (0, 9) is on the graph, and 0 is 3 units to the right of the axis of symmetry.

The point that is 3 units to the left of the axis is –6, so the point (–6, 9) is also on the graph.

Determine two additional points on the graph.

Choose an x-value to the left or right of the vertex and find the corresponding y-value.

$f(x) = x^2 + 6x + 9$	Original equation
$f(-1) = (-1)^2 + 6(-1) + 9$	Substitute –1 for x.
$f(-1) = 4$	Simplify.

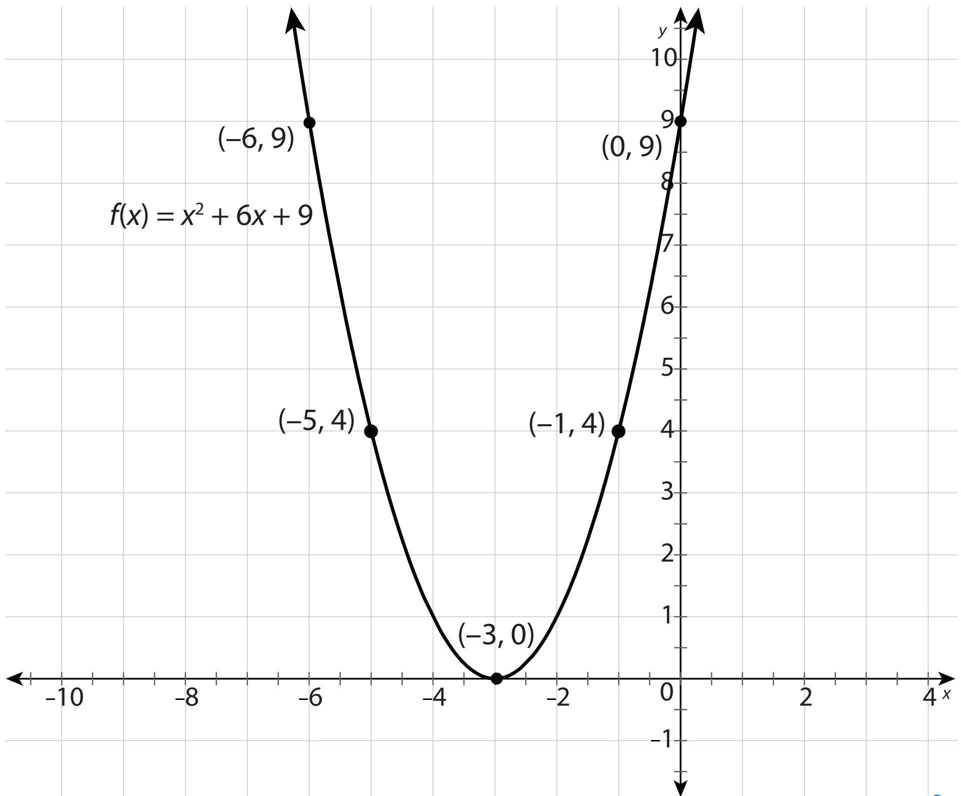
An additional point is (–1, 4).

(–1, 4) is 2 units to right of the axis of symmetry.

The point that is 2 units to the left of the axis is –5, so the point (–5, 4) is also on the graph.

(continued)

Plot the points and join them with a smooth curve.



Example 4

Given the function $f(x) = -2x^2 - 12x - 10$, identify the key features of its graph: the extremum, vertex, and y-intercept. Then sketch the graph.

1. Determine the extremum of the graph.

   The extreme value is either a minimum, when $a > 0$, or a maximum, when $a < 0$.

   Because $a = -2$, the graph opens down and the quadratic has a maximum.

2. Determine the vertex of the graph.

The vertex is of the form $\left(\frac{-b}{2a}, f\left(\frac{-b}{2a}\right)\right)$.

Use the original function $f(x) = -2x^2 - 12x - 10$ to find the values of a and b in order to find the x-coordinate of the vertex.

$x = \frac{-b}{2a}$	Formula to find the x-coordinate of the vertex of a parabola
$x = \frac{-(-12)}{2(-2)}$	Substitute –2 for a and –12 for b.
$x = -3$	Simplify.

The x-coordinate of the vertex is –3.

Substitute –3 into the original equation to find the y-coordinate.

$f(x) = -2x^2 - 12x - 10$	Original equation
$f(-3) = -2(-3)^2 - 12(-3) - 10$	Substitute –3 for x.
$f(-3) = 8$	Simplify.

The y-coordinate of the vertex is 8.

The vertex is (–3, 8).

3. Determine the y-intercept of the graph.

The y-intercept occurs when $x = 0$.

Substitute 0 for x in the original equation.

$f(x) = -2x^2 - 12x - 10$	Original equation
$f(0) = -2(0)^2 - 12(0) - 10$	Substitute 0 for x.
$f(0) = -10$	Simplify.

The y-intercept is (0, –10).

4. Graph the function.

   Use symmetry to identify another point on the graph.

   Because 0 is 3 units to the right of the axis of symmetry, the point 3 units to the left of the axis will have the same value, so (–6, –10) is also on the graph.

   Determine two additional points on the graph.

   Choose an *x*-value to the left or right of the vertex and find the corresponding *y*-value.

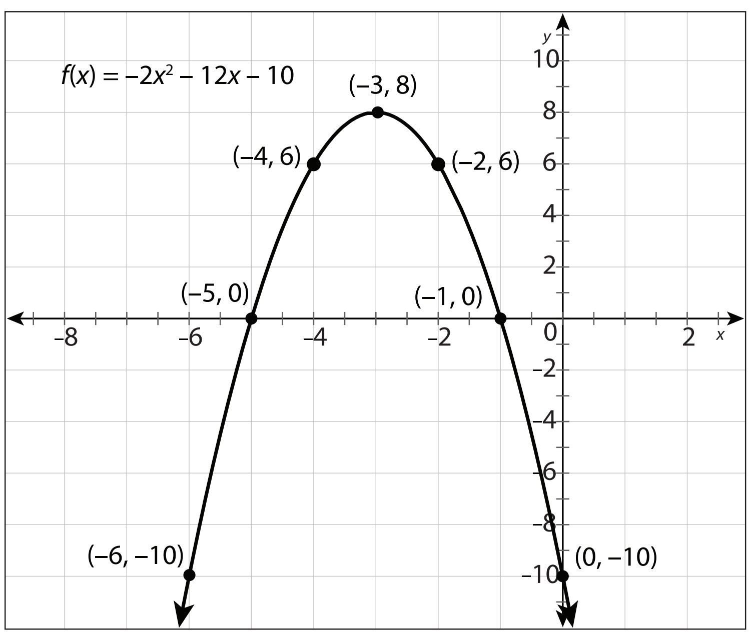
$f(x) = -2x^2 - 12x - 10$	Original equation
$f(0) = -2(-2)^2 - 12(-2) - 10$	Substitute –2 for *x*.
$f(-2) = 6$	Simplify.

   An additional point is (–2, 6).

   (–2, 6) is 1 unit to right of the axis of symmetry.

   The point that is 1 unit to the left of the axis is –4, so the point (–4, 6) is also on the graph.

   Plot the points and join them with a smooth curve.



Example 5

$h(x) = 2x^2 - 11x + 5$ is a quadratic function. Determine the direction in which the function opens, the coordinates of the vertex, the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

1. Determine whether the graph opens up or down.

   $h(x) = 2x^2 - 11x + 5$ is in standard form; therefore, $a = 2$.

   Since $a > 0$, the parabola opens up.

2. Find the vertex and the equation of the axis of symmetry.

   $h(x) = 2x^2 - 11x + 5$ is in standard form; therefore, $a = 2$ and $b = -11$.

$x = \dfrac{-b}{2a}$	The x-coordinate of the vertex
$x = \dfrac{-(-11)}{2(2)}$	Substitute 2 for a and –11 for b.
$x = 2.75$	Simplify.

   The vertex has an x-coordinate of 2.75.

   Since the input value is 2.75, find the output value by evaluating the function for $x = 2.75$.

$h(x) = 2x^2 - 11x + 5$	Original equation
$h(2.75) = 2(2.75)^2 - 11(2.75) + 5$	Substitute 2.75 for x.
$h(2.75) = -10.125$	Simplify.

   The y-coordinate of the vertex is –10.125.

   The vertex is the point (2.75, –10.125).

   Since the axis of symmetry is the vertical line through the vertex, the equation of the axis of symmetry is $x = 2.75$.

3. Find the y-intercept.

   $h(x) = 2x^2 - 11x + 5$ is in standard form, so the y-intercept is the constant term c, which is 5.

   The y-intercept is 5.

4. Find the x-intercepts, if any exist.

   The x-intercepts occur when $y = 0$.

   Substitute 0 for the output, $h(x)$, and solve.

   This equation is factorable, but if we cannot easily identify the factors, the quadratic formula always works.

   Note both methods.

Solved by factoring:

$h(x) = 2x^2 - 11x + 5$

$0 = 2x^2 - 11x + 5$

$0 = (2x - 1)(x - 5)$

$0 = 2x - 1$ or $0 = x - 5$

$x = 0.5$ or $x = 5$

Solved using the quadratic formula:

$h(x) = 2x^2 - 11x + 5$

$0 = 2x^2 - 11x + 5$

$a = 2$, $b = -11$, and $c = 5$

$$x = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x = \frac{-(-11) \pm \sqrt{(-11)^2 - 4(2)(5)}}{2(2)}$$

$$x = \frac{11 \pm \sqrt{81}}{4}$$

$$x = \frac{11 \pm 9}{4}$$

$$x = \frac{11 - 9}{4} \text{ or } x = \frac{11 + 9}{4}$$

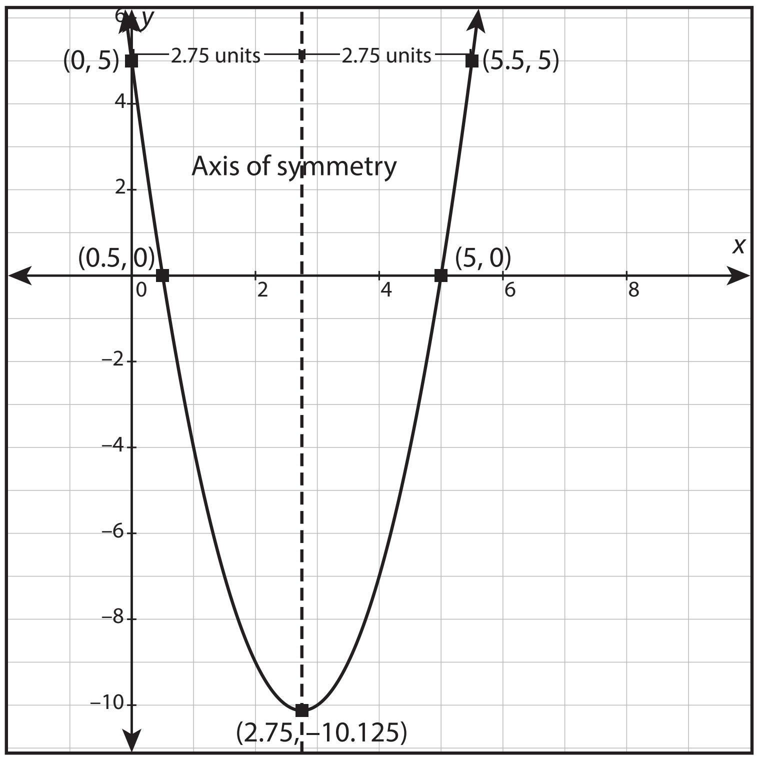
$$x = \frac{2}{4} \text{ or } x = \frac{20}{4}$$

$x = 0.5$ or $x = 5$

The x-intercepts are 0.5 and 5.

5. Plot the points from steps 2–4 and their symmetric points over the axis of symmetry.

   Connect the points with a smooth curve.



Example 6

$g(x) = -x^2 + 8x - 17$ is a quadratic function. Determine the direction in which the function opens, the vertex, the equation of the axis of symmetry, the x-intercept(s), if any, and the y-intercept. Use this information to sketch the graph.

1. Determine whether the graph opens up or down.

   $g(x) = -x^2 + 8x - 17$ is written in standard form; therefore, $a = -1$.

   Since $a < 0$, the parabola opens down.

2. Find the vertex and the equation of the axis of symmetry.

   $g(x) = -x^2 + 8x - 17$ is written in standard form; therefore, $a = -1$ and $b = 8$.

$x=\dfrac{-b}{2a}$	The x-coordinate of the vertex
$x=\dfrac{-(8)}{2(-1)}$	Substitute –1 for a and 8 for b.
$x = 4$	Simplify.

   The vertex has an x-coordinate of 4.

   Since the input value is 4, find the output value by evaluating the function for $x = 4$.

$g(x) = -x^2 + 8x - 17$	Original equation
$g(4) = -(4)^2 + 8(4) - 17$	Substitute 4 for x.
$g(4) = -1$	Simplify.

   The y-coordinate of the vertex is –1.

   The vertex is the point (4, –1).

   Since the axis of symmetry is the vertical line through the vertex, the equation of the axis of symmetry is $x = 4$.

3. Find the y-intercept.

   The function $g(x) = -x^2 + 8x - 17$ is in standard form, so the y-intercept is the constant term c, which equals –17.

   The y-intercept is –17.

4. Find the x-intercepts, if any exist.

   The x-intercepts occur when $y = 0$.

   Substitute 0 for the output, $g(x)$, and solve using the quadratic formula since the function is not factorable over the rational numbers.

$g(x) = -x^2 + 8x - 17$	Original equation
$-x^2 + 8x - 17 = 0$	Set the equation equal to 0.

   Determine the values of a, b, and c.

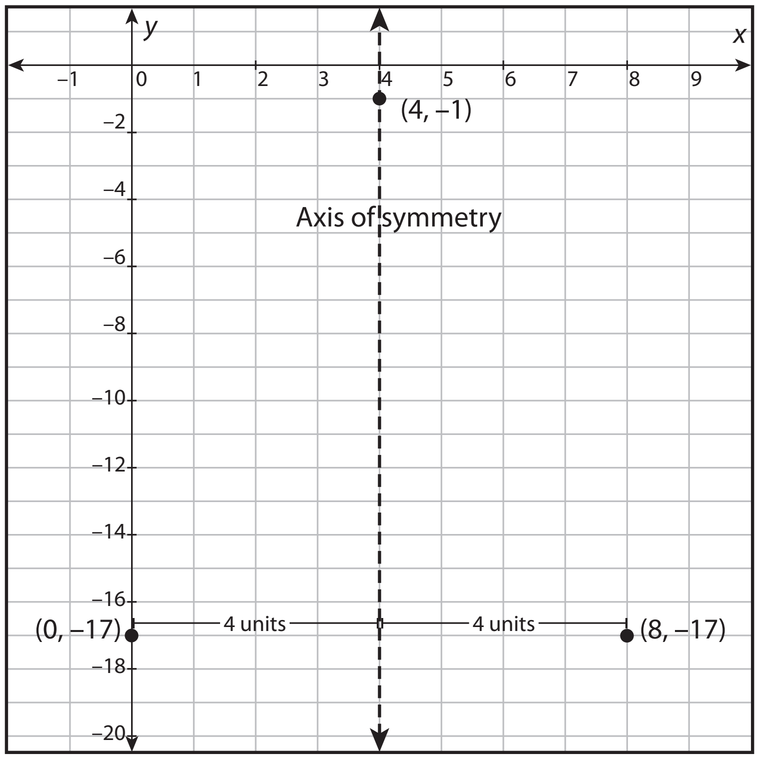
   $a = -1$, $b = 8$, and $c = -17$

$x=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}$	Quadratic formula
$x=\dfrac{-(8)\pm\sqrt{(8)^2-4(-1)(-17)}}{2(-1)}$	Substitute –1 for a, 8 for b, and –17 for c.
$x=\dfrac{-8\pm\sqrt{-4}}{-2}$	Simplify.

   In this case, the discriminant, –4, is negative, which means there are no real solutions.

   This also means that there are no x-intercepts.

5. Plot the points from steps 2–4 and their symmetric points over the axis of symmetry.



For a more accurate graph, determine an additional pair of symmetric points.

Choose any x-value on the left or right of the axis of symmetry.

Evaluate the function for the chosen value of x to determine the output value.

Let's choose $x = 1$.

$g(x) = -x^2 + 8x - 17$	Original equation
$g(1) = -(1)^2 + 8(1) - 17$	Substitute 1 for x.
$g(1) = -10$	Simplify.

(1, –10) is an additional point on the parabola.

Plot (1, –10) on the same graph.

(1, –10) is 3 units from the axis of symmetry.

Locate the point that is symmetric to the point (1, –10) with respect to the axis of symmetry.

(*continued*)

(7, –10) is also 3 units from the axis of symmetry and is symmetrical to the original point (1, –10) with respect to the axis of symmetry.

You can verify that (1, –10) and (7, –10) are the same distance from the axis of symmetry and are also symmetrical by referring to the graph.