

Lesson 2.5: Defining Complex Numbers,  $i$ , and  $i^2$ 

## Instruction

## Guided Practice 2.5

## Example 1

Identify the real and imaginary parts of the complex number  $8 + \frac{1}{3}i$ .

1. Identify the real part of the complex number.

Identify the part that is not a multiple of  $i$ .

8 is not a multiple of  $i$ .

The real part of  $8 + \frac{1}{3}i$  is 8.



2. Identify the imaginary part of the complex number.

Identify the part that is a multiple of  $i$ .

$\frac{1}{3}$  is a multiple of  $i$ .

The imaginary part of  $8 + \frac{1}{3}i$  is the term  $\frac{1}{3}i$ .



## Example 2

Rewrite the complex number  $2i$  using a radical.

1. Replace  $i$  with  $\sqrt{-1}$ .

$$2i = 2 \cdot \sqrt{-1}$$



2. Place the squared value of any whole multiples of  $i$  under the radical sign.

$$2 \cdot \sqrt{-1} = \sqrt{2^2 \cdot (-1)}$$



3. Simplify the radical expression.

$$\sqrt{2^2 \cdot (-1)} = \sqrt{4 \cdot (-1)} = \sqrt{-4}$$



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**Example 3**

Rewrite the radical  $\sqrt{-32}$  using the imaginary unit  $i$ .

1. Rewrite the radicand as the product of  $-1$  and a positive value.

$$\sqrt{-32} = \sqrt{(-1) \cdot 32}$$

2. Rewrite the radical  $\sqrt{-1}$  as  $i$ .

$$\sqrt{(-1) \cdot 32} = \sqrt{-1} \cdot \sqrt{32} = i \cdot \sqrt{32}$$

3. If possible, rewrite the real number radicand as the product of two factors, where one factor is the largest perfect square factor of the radicand.

$32 = 16 \cdot 2$ , and  $16$  is the factor of  $32$  that is the largest perfect square.

$$i \cdot \sqrt{32} = i \cdot \sqrt{16 \cdot 2}$$

4. Rewrite the radicand as the product of two roots, and simplify the square root of the perfect square. Then, write the result as the coefficient of  $i$ .

$$i \cdot \sqrt{16 \cdot 2} = i \cdot \sqrt{4^2 \cdot 2} = 4i\sqrt{2}$$

**Example 4**

Simplify  $i^{57}$ .

1. Find the remainder of the power of  $i$  when divided by 4.

$$14 \cdot 4 = 56; \text{ therefore, } 57 \div 4 = 14 \text{ remainder } 1.$$

The remainder is 1.

2. Use the remainder to simplify the power of  $i$ .

$$i^{57} = (i^4)^{14} \cdot i^1 = 1 \cdot i^1 = i$$

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**Example 5**

Impedance,  $Z$ , is the measure of a circuit's opposition to the flow of current. Complex numbers are used to represent the impedance of a circuit. The resistance,  $R$ , is the real part of the impedance, and the reactance,  $X$ , is the coefficient of the imaginary unit  $i$ . So, impedance is  $R + Xi$ , where  $R$  and  $X$  are both measured in ohms. A certain circuit has a resistance of 18 ohms and a reactance of 2 ohms. Use a complex number to represent the circuit's impedance.

1. Use the resistance to write the real part of the complex number.

The resistance is 18 ohms; therefore, the real part of the number is 18.



2. Use the reactance to write the imaginary part of the complex number.

The reactance is the coefficient of  $i$ .

The reactance is 2 ohms; therefore, the coefficient of  $i$  is 2.

The imaginary part of the number is  $2i$ .



3. The complex representation of impedance is the sum of the real and imaginary parts.

The circuit's impedance in ohms is  $18 + 2i$ .

